12. Homework Assignment

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 1: Consider the parameter-dependent vector field

$$\begin{aligned} \dot{x} &= 2(x+y)^2 + x - y - \lambda, \\ \dot{y} &= -x + y - \lambda. \end{aligned}$$

Sketch the flow for small $|\lambda|$ and justify your claims.

Problem 2: Let $\mathcal{M}^c = \operatorname{graph}\psi$ be a $\mathcal{C}^1$ center manifold of the flow ϕ^t to the vector field

$$\dot{x} = Ax + g(x)$$

with $g(0) = 0,\ Dg(0) = 0$.
Here $\psi : E^c \to E^h$ where $\mathbb{R}^N = E^c \oplus E^h$ is the eigenspace decomposition w.r.t. A.

Prove that $\mathcal{M}^c$ is tangential to E^c, i.e. $\psi'(0) = 0$.

Hint: Use the invariance of $\mathcal{M}^c$ under the flow ϕ^t.

Problem 3: Consider the set M of matrices defined by

$$M := \left\{ \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}, a, b, c \in \mathbb{R} \right\}.$$

Show, that for each $A \in M$ the set $\left\{\exp(A^T t), t \in \mathbb{R}\right\}$ is a subgroup of $SO(3)$. Describe this group. $SO(3)$ denotes the set of orthogonal matrices with determinant one.

Problem 4: Consider the Lie group $SL_2(\mathbb{R})$, i.e. the set of real 2×2 matrices with determinant one.

(i) Show, that the Lie algebra $\mathfrak{sl}_2(\mathbb{R})$, that is the tangent space of $SL_2(\mathbb{R})$ at Id is the space of real 2×2 matrices with trace zero.

(ii) Is the matrix exponential $\exp : \mathfrak{sl}_2(\mathbb{R}) \to SL_2(\mathbb{R})$ surjective?